Math 127 **Final Review** **Name**_____ _____

No calculator, unless marked with an asterisk (*)

Final Exam Formulas			
$\vec{v} = \|\vec{v}\|cos\theta \cdot \boldsymbol{i} + \|\vec{v}\|sin\theta \cdot \boldsymbol{j}$	$\cos\theta = \frac{v \cdot w}{\|v\|\|w\|}$	$Proj_w v = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2}(\vec{w})$	$\boldsymbol{v}_2 = \boldsymbol{v} - \boldsymbol{v}_1$

1) Write the equation of an ellipse in standard form that meets the requirements below:
foci: $(0,-2),(0,2)$; y-intercepts: -5 and 5.

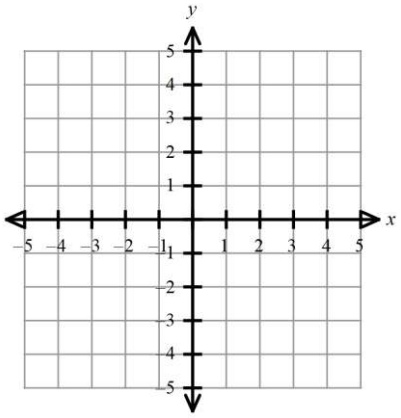
2) Write the equation of a hyperbola in standard form that meets the requirements below:
foci: $(0,-4),(0,4)$; vertices: $(0,-3),(0,3)$.

For #3 – 5, graph the conic and find the requested information. If needed, round to 3 decimal places.

3) $4x^2 + 8x + 9y^2 = 32$ (Convert to standard form by completing the square)

Center:

Foci:



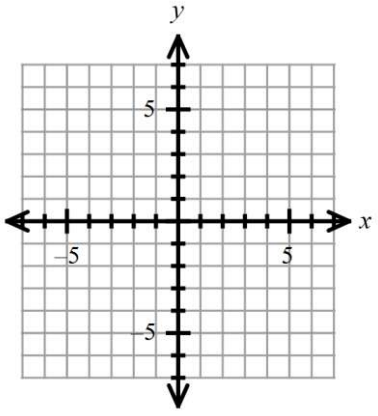
4) $x^2 - 2x + 7y - 34 = 0$ (Convert to standard form by completing the square)

Vertex:

Focus:

Directrix:

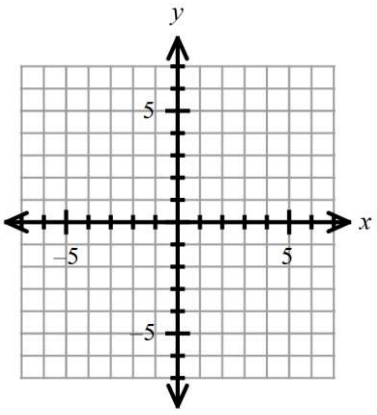
Length of
Latus Rectum:



5) $\frac{(x-2)^2}{16} - \frac{(y+2)^2}{4} = 1$

Center:

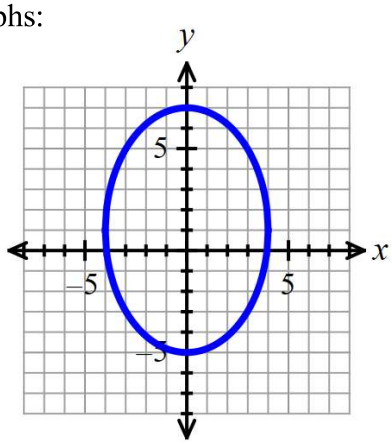
Foci:



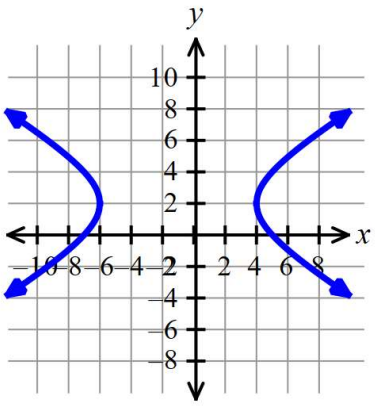
For #6 – 11, match each graph to its equation. No item will be used more than once. Not all equations will be used.

Graphs:

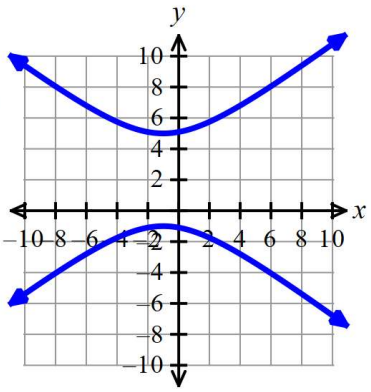
6)



7)



8)



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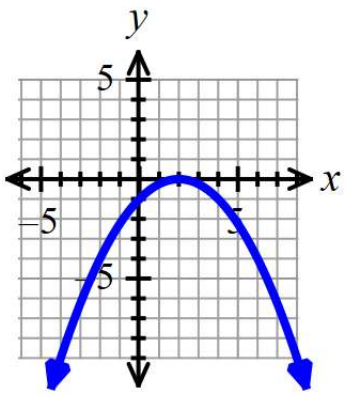

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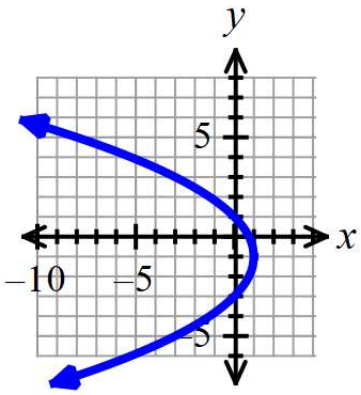
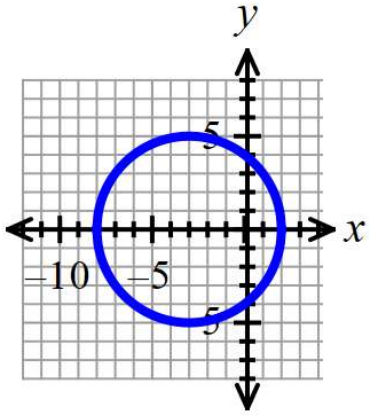


11)



Equations:

A) $\frac{(y-2)^2}{9} - \frac{(x+1)^2}{16} = 1$

B) $\frac{(x+1)^2}{25} - \frac{(y-2)^2}{9} = 1$

C) $(y+1)^2 = -4(x-1)$

D) $\frac{x^2}{16} + \frac{(y-1)^2}{36} = 1$

E) $\frac{x^2}{36} + \frac{(y+1)^2}{16} = 1$

F) $(x-2)^2 = -4y$

G) $(x-3)^2 + y^2 = 25$

H) $(x+3)^2 + y^2 = 25$

12) Write the equation of a parabola in standard form that meets the requirements below:
Focus at $(3,-2)$ and directrix at $y = 0$.

For 13 – 16 , verify the trig identity:

13) $\tan x(\cot x - \cos x) = 1 - \sin x$

14) $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

15) $\frac{1+\cos 2x}{\sin 2x} = \cot x$

16) $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

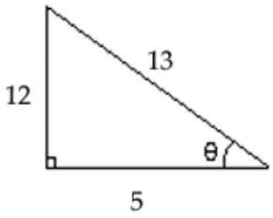
17) Verify the identity: $\tan^2\theta + 4 = \sec^2\theta + 3$

For 18 – 19: Find the exact value by using a sum or difference identity (Hint: consider using a sum or difference identity).

18) $\tan\frac{7\pi}{12}$

19) $\cos\frac{11\pi}{12}$

20) Use the figure to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.



21) Use the given information to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.
$\sin\theta = \frac{4}{5}$, θ lies in quadrant I

For 22 – 23, find all solutions of the following equations.

22) $2\sin x - \sqrt{3} = 0$

23) $\tan x \sec x = -2\tan x$

For 24 – 27, solve the equation on the interval $[0, 2\pi)$.

24) $\cos 2x = \frac{\sqrt{3}}{2}$

25) $\cos^2 x + 2\cos x + 1 = 0$

26) $\cos x + 2\cos x \sin x = 0$

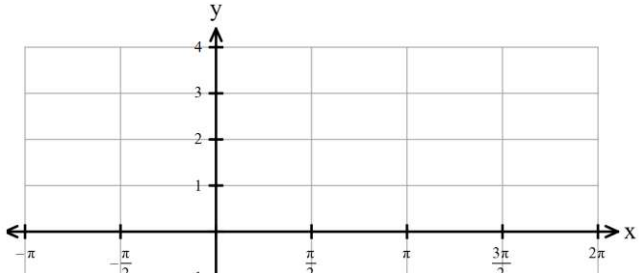
27) $\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 1$

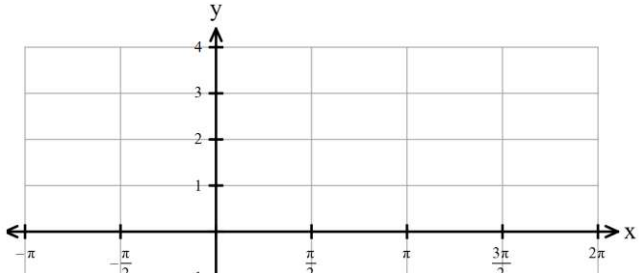
28) Given $\sin\alpha = \frac{3}{7}$, α lies in quadrant II, $\cos\beta = \frac{5}{13}$, and β lies in quadrant I. Find $\sin(\alpha - \beta)$.

For 29 – 36, graph the function on the provided coordinate plain.

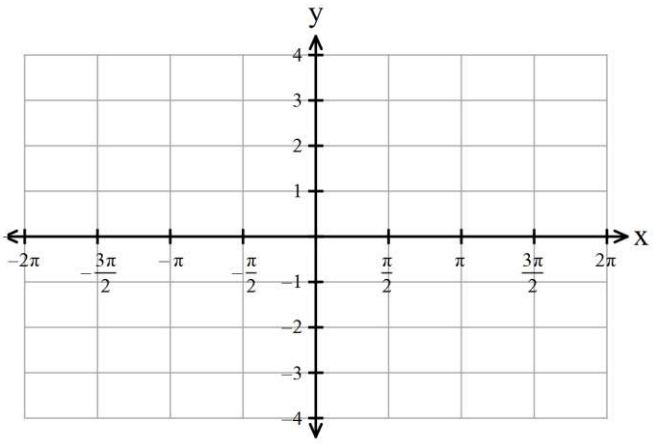
29) $y = 3\sin(2x)$

30) $y = -2\sin\left(x + \frac{\pi}{4}\right) - 1$

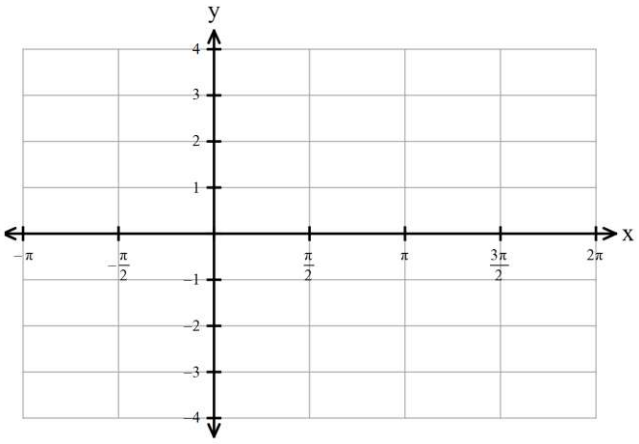




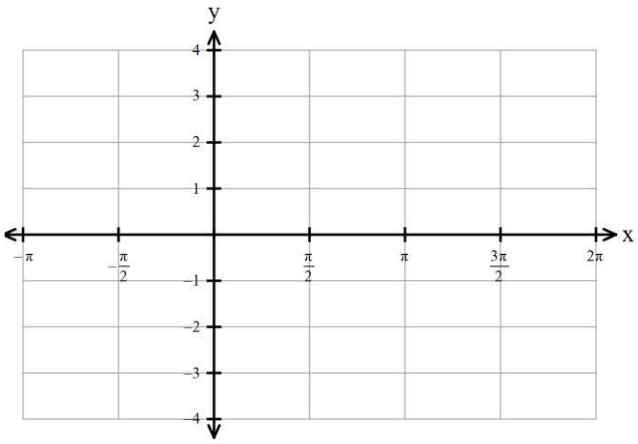
31) $y = 3\cos\left(\frac{1}{2}x\right)$



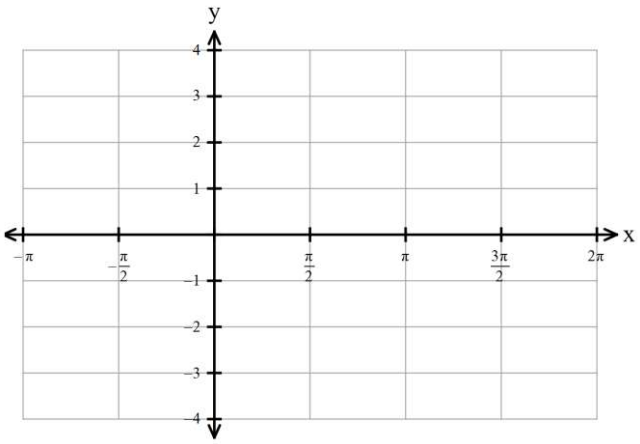
32) $y = -3\cos(2x - \pi)$



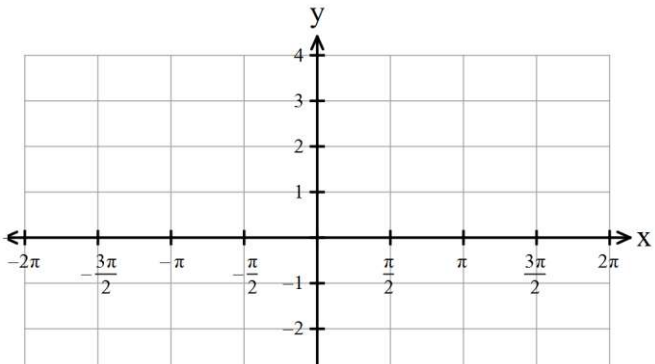
33) $y = \tan(2x)$



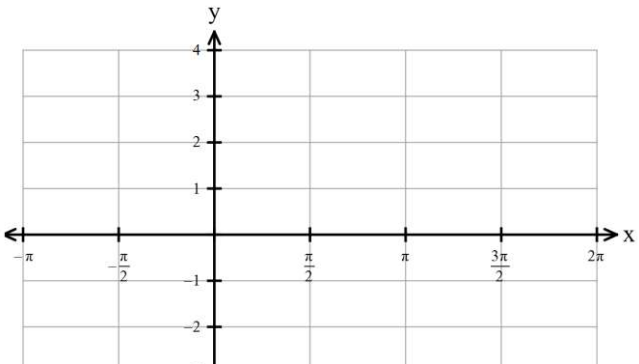
34) $y = 4\cot\left(\frac{x}{2}\right)$



35) $y = -\sec\frac{1}{2}x$



36) $y = 2\csc(x + \frac{\pi}{2})$



37) Describe the transformations and period for $y = -4\cos(3x + 1) - 3$.

For 38 – 41, find the exact value of the expression, use the restricted range.

38) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ 39) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 40) $\cos^{-1}(1)$ 41) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

For 42 – 47, find the value of the expression. Use a sketch but do NOT use a calculator.

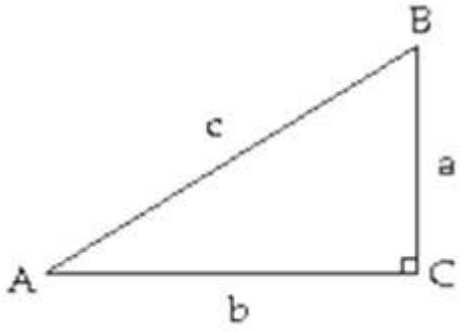
42) $\tan^{-1}[\tan\left(\frac{\pi}{5}\right)]$ 43) $\cot[\sin^{-1}\left(\frac{5}{7}\right)]$

44) $\cos(\tan^{-1} x);\ x > 0$ 45) $\sin[\sec^{-1}\left(\frac{\sqrt{x^2+9}}{x}\right)];\ x > 0$

46) $\cos[\sin^{-1}\left(-\frac{3}{5}\right)]$ 47) $\sin^{-1}[\sin\left(\frac{13\pi}{7}\right)]$

***For 48 – 49, solve the right triangle shown in the figure. Round all answers to the nearest tenth.**

48) $a = 3.8\ cm, b = 2.4\ cm$



49) $a = 3.3\ in, A = 55.1°$

For 50 – 51, convert the following coordinates from either polar to rectangular or rectangular to polar.

50) $(-3, 120°)$

51) $(2\sqrt{3}, 2)$ (answer in radians)

For 52 – 53, convert the following equations from either polar to rectangular or rectangular to polar.

52) $(x-16)^2 + y^2 = 256$

53) $r = 4\csc\theta$

*54) Write the complex number, $-6 + 8i$, in polar form.

55) Write the complex number, $3(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$, in rectangular form.

56) Find the product of the complex numbers.
$z_1 = 5(\cos 200° + i\sin 200°)$
$z_2 = 4(\cos 50° + i\sin 50°)$

57) Find the quotient of the complex numbers.
$z_1 = 5(\cos 20° + i\sin 20°)$
$z_2 = 4(\cos 10° + i\sin 10°)$

58) Use DeMoivre's Theorem to find the indicated power of the complex number. Write the answer in rectangular form. $\left[10\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]^3\right.$

59) Given $\boldsymbol{u}=-12i-2j, \boldsymbol{v}=6i-7j$; Find $3\boldsymbol{u}-\boldsymbol{6v}$.

*60) Given $\vec{v}=10i-4j$; Find $\|-7\vec{v}\|$. Round answer to the nearest tenth.

*61) Given that $\vec{v}=-3\boldsymbol{i}-4\boldsymbol{j}$ and $\vec{w}=6\boldsymbol{i}+8\boldsymbol{j}$. Find the measure of the angle (in degrees) between the vectors, and classify the vectors as parallel, orthogonal, or neither.

62) Given $\vec{v}$ with $\|\vec{v}\| = 7$ and $\theta = 225°$. Write the vector $\vec{v}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

*63) Two airplanes leave an airport at the same time on different runways. One flies S 15° E at 250 mph. The other airplane flies on a bearing of N 63° E at 302 mph. How far apart will the airplanes be after 2.5 hours? Round to the nearest mile.

64) Find the unit vector of the following vector, $\vec{v} = 3\boldsymbol{i} - 4\boldsymbol{j}$.

65) Write an equivalent representation in the form (r, θ) for the point $\left(2, \frac{5\pi}{6}\right)$, where $r < 0$ and $-2\pi < \theta < 0$?

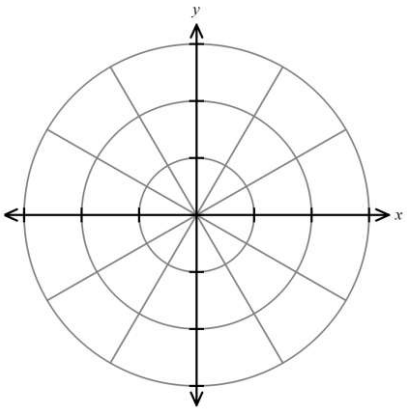
*66) Identify the number of possible triangles. Then solve for all missing sides and angles. Round all values to the nearest tenth. b = 15, c = 17, B = 42°

Answers
66) # of Δs: _____
A_1 = _____ A_2 = _____
C_1 = _____ C_2 = _____
a_1 = _____ a_2 = _____

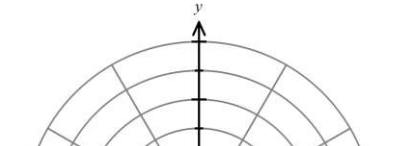
For 67- 71, graph each polar curve. An optional table is available for you to use, if desired.

67) Graph $r = 3 \sin \theta$.

θ	r



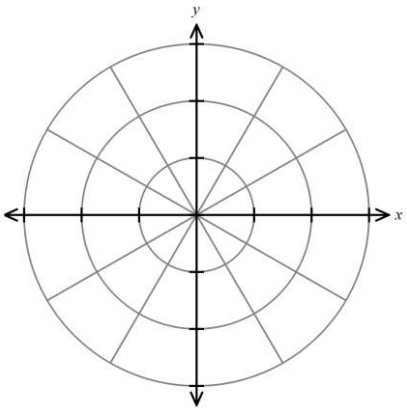
68) Graph $r = 3 + 3 \sin \theta$.



θ	r

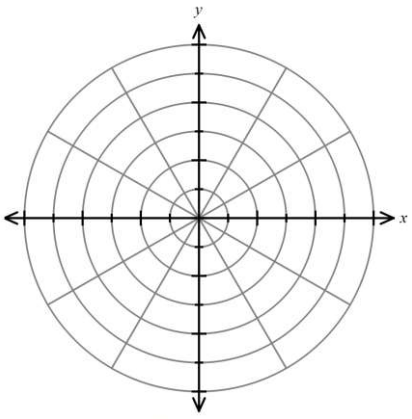
69) Graph $r = 3 \sin 2\theta$.

θ	r



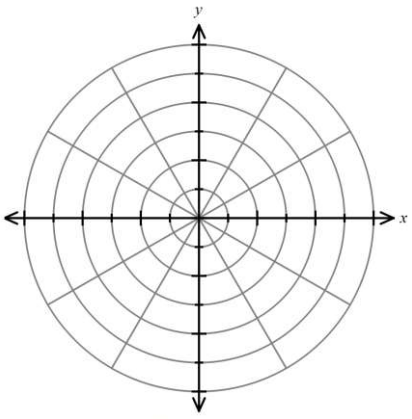
70) Graph $r = 2 - 3 \cos \theta$.

θ	r



71) Graph $r = 3 + 2 \sin \theta$.

θ	r



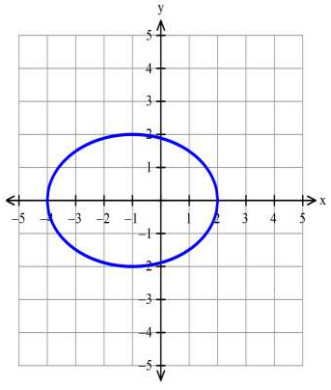
*72) A hiker leaves the trailhead with a bearing of N 42° W. After traveling 3.2 miles, the hiker then turns 90° and travels on a bearing of S 48° W for 2.4 miles. At that time, what is the bearing of the hiker from the trailhead? Round to the nearest tenth.

*73) A boat is in the ocean, and a nearby cliff has a restaurant on its edge. The angle of elevation from the boat to the top of the restaurant is 67 degrees, and the angle of elevation of the boat to the bottom of the restaurant is 48 degrees. If the boat is 50 feet away from the base of the cliff at sea level, then find the height of the restaurant, rounded to the nearest foot.

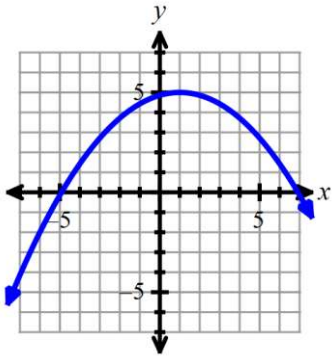
Final Review Key:

1) $\frac{x^2}{21}+\frac{y^2}{25}=1$ 2) $\frac{y^2}{9}-\frac{x^2}{7}=1$

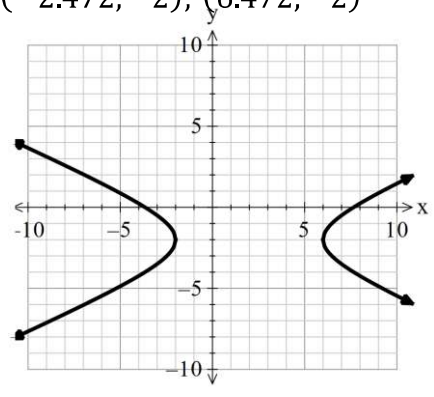
3) C: (-1, 0)
F: $(-3.236,0),(1.236,0)$



4) V: (1, 5) F: (1,3.25)
D: $y=6.75$ Length of LR: 7



5) C: (2, -2);
F: $(-2.472,-2);\ (6.472,-2)$



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6) D 7) B 8) A 9) F 10) C 11) H 12) $(x-3)^2 = -4(y+2)$ 13 – 17) Possible solutions:

13) $\tan x\,(\cot x - \cos x) = 1 - \sin x$

$\tan x \cdot \cot x - \tan x \cos x$

$\frac{\sin x}{\cos x}\cdot\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\cdot\cos x$

$1 - \sin x = 1 - \sin x$ ✓

14) $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$\tan\alpha + \tan\beta = \tan\alpha + \tan\beta$ ✓

15) $\frac{1+\cos 2x}{\sin 2x} = \cot x$ $\quad 1-\sin^2 x = \cos^2 x$

$\frac{1+\cos^2 x - \sin^2 x}{2\sin x\cos x}$

$\frac{\cos^2 x + \cos^2 x}{2\sin x\cos x}$

$\frac{2\cos^2 x}{2\sin x\cos x}$

$\frac{\cos x}{\sin x}$

$\cot x = \cot x$ ✓

16) $\sin\left(x+\frac{\pi}{2}\right) = \cos x$

$\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2}$

$\sin x\cdot 0 + \cos x\cdot 1$

$\cos x = \cos x$ ✓

17) Verify the identity: $\tan^2\theta + 4 = \sec^2\theta + 3$

$\sec^2\theta - 1 + 4$

$\sec^2\theta + 3 = \sec^2\theta + 3$ ✓

18) $-2-\sqrt{3}$ 19) $\frac{-\sqrt{2}-\sqrt{6}}{4}$ 20) $\frac{120}{169}; -\frac{119}{169}; -\frac{120}{119}$ 21) $\frac{24}{25}; -\frac{7}{25}; -\frac{24}{7}$ 22) $x = \frac{\pi}{3} + 2\pi n$ or $x = \frac{2\pi}{3} + 2\pi n$

23) $x = \frac{2\pi}{3} + 2\pi n$ or $x = \frac{4\pi}{3} + 2\pi n$ or $x = \pi n$ 24) $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$ 25) π

26) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ 27) 0 28) $\frac{15+24\sqrt{10}}{91}$

Graphs for 29-36

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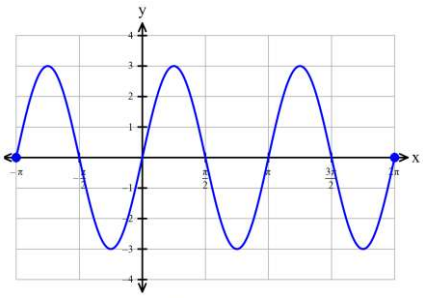
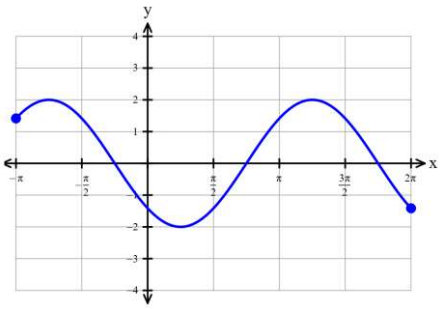
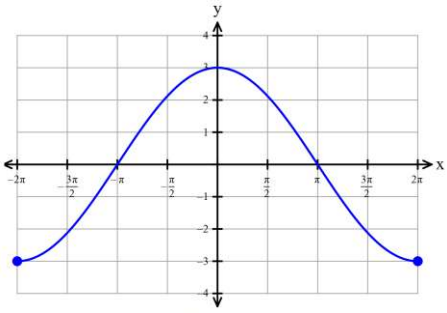
29) $y = 3\sin 2x$



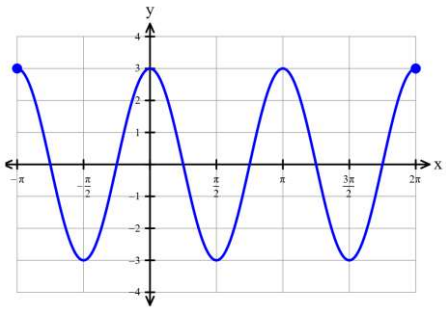
30) $y = -2\sin(x + \frac{\pi}{4}) - 1$



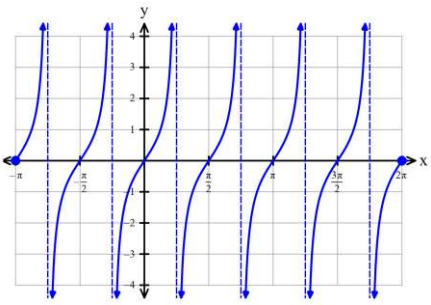
31) $y = 3\cos\frac{1}{2}x$



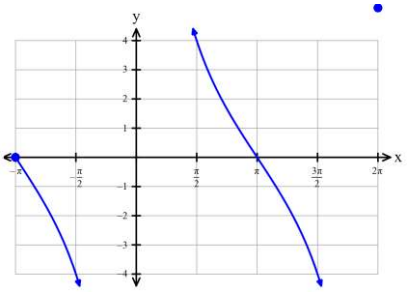
32) $y = -3\cos(2x - \pi)$



33) $y = \tan 2x$

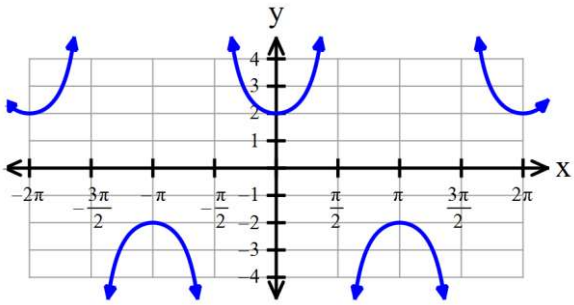


34) $y = 4\cot\left(\frac{x}{2}\right)$



35) $y = -\sec\left(\frac{1}{2}x\right)$

36) $y = 2\csc\left(x + \frac{\pi}{2}\right)$



37) Vert. Ref, Vert. Stretched by 4, Period $= \frac{2\pi}{3}$, Phase Shift $= -\frac{\pi}{3}$, Vert.Shift $= -3$

38) $-\frac{\pi}{3}$ 39) $\frac{3\pi}{4}$ 40) 0 41) $\frac{\pi}{6}$ 42) $\frac{\pi}{5}$ 43) $\frac{2\sqrt{6}}{5}$ 44) $\frac{\sqrt{x^2+1}}{x^2+1}$ 45) $\frac{9\sqrt{x^2+9}}{x^2+9}$ 46) $\frac{4}{5}$

47) $-\frac{\pi}{7}$ 48) $c = 4.5; A = 40.2°; B = 28.1°$ 49) $c = 4.0; b = 2.3; B = 34.9°$ 50) $(\frac{3}{2}, -\frac{3\sqrt{3}}{2})$

51) $(4, \frac{\pi}{6})$ 52) $r = 32\cos\theta$ 53) $y = 4$ 54) $10\ cis\ 126.9°$ 55) $-\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

56) $20\ cis\ 250°$ 57) $\frac{5}{4}\ cis\ 10°$ 58) $1000\ cis\frac{9\pi}{4}$ 59) $-72i + 36j$ 60) 75.4

61) 180°; parallel 62) $-\frac{7\sqrt{2}}{2}i - \frac{7\sqrt{2}}{2}j$ 63) 1076 miles 64) $\frac{3}{5}i - \frac{4}{5}j$ 65) $(-2, -\frac{\pi}{6})$

Answers

66)

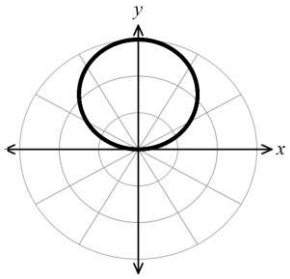
33) # of Δs: ___2_____

$A_1 = 88.7°$ $A_2 = 7.3°$

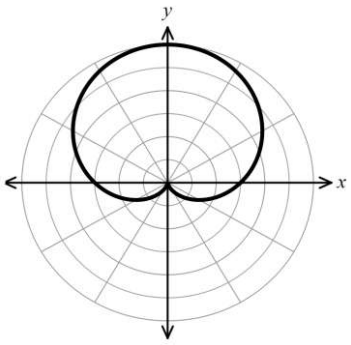
$C_1 = 49.3°$ $C_2 = 130.7°$

$a_1 = 22.4$ $a_2 = 2.8$

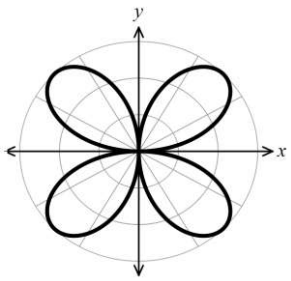
67)



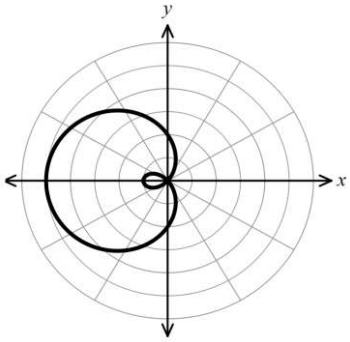
68)



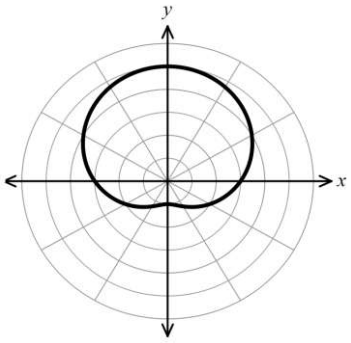
69)



70)



71)



72) $N\ 78.9°\ W$

73) $62\ ft.$